PHYC 581: High Energy Astrophysics

Fall 2018

Lecture 13: 10/03/2018

Compton Scattering:

In the Compton scattering process, the incoming high-energy photons collide with stationary electrons and transfer some of their energy and momentum to the electrons. Consequently, the photons come out of the process with less energy and momentum.

Let us begin with a simplified classical treatment of this process. An electron subject to the electromagnetic field of an incoming photon accelerates at a rate $a = \frac{e}{m_e} E$, with E being the electric field, which results in a radiated power:

$$P = \frac{2}{3} \frac{e^4}{c^3 m_e^2} E^2$$

The incident energy flux is given by $f = c u_{rad}$, where u_{rad} is the radiation energy density: $u_{rad} = \frac{E^2}{4\pi}$. The scattering cross section then follows:

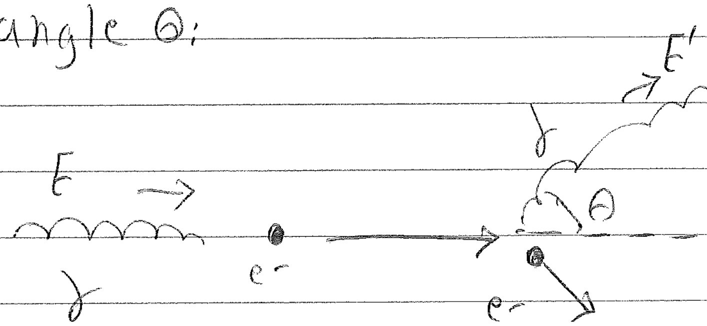
$$\sigma \equiv \frac{P}{f} = \frac{8\pi}{3} r_e^2 \qquad \left(r_e \equiv \frac{e^2}{m_e c^2}\text{; classical radius of the electron}\right)$$

This is the "Thomson" scattering cross section:

$$\sigma_T = \frac{8\pi}{3} r_e^2 \approx \frac{2}{3} \times 10^{-24}\ \text{cm}^2$$

When the incoming photon is very energetic, the recoil of the electron cannot be ignored. The energy of the scattered photon depends on the scattering angle θ:

$$E' = \frac{E}{1 + \frac{E}{m_e c^2}(1 - \cos\theta)}$$



For the forward scattering, $\theta = 0$, we have $E' = E$. While, $E' < E$ for all other values of θ. We note that in the limit $E \ll m_e c^2$, $E' \simeq E$ regardless of the value of θ.

The total cross section for Compton scattering can be found by using a full quantum treatment withi quantum electrodynamics (QED), and is given by the Klein-Nishina formula:

$$\sigma_{K-N} = \pi r_e^2 \frac{1}{\varepsilon}\left[\left(1-\frac{2(1+\varepsilon)}{\varepsilon^2}\right)\ln(2\varepsilon+1)+\frac{1}{2}+\frac{4}{\varepsilon}-\frac{1}{2(2\varepsilon+1)^2}\right]$$

Here, $\varepsilon = \frac{E}{m_e c^2}$. In the non-relativistic limit, $E \ll m_e c^2$, this is reduced to Thomson cross section:

$$\sigma_{K-N} \simeq \frac{8\pi}{3} r_e^2 (1-2\varepsilon) \simeq \sigma_T \qquad \varepsilon \ll 1$$

In the relativistic limit, $\varepsilon \gg 1$ we have:

$$\sigma_{K-N} \approx \frac{\pi r_e^2}{\varepsilon}\left(\ln 2\varepsilon + \frac{1}{2}\right) \qquad \varepsilon \gg 1$$

The scattering cross section can be used to calculate the power emitted by an energetic electron moving through photons. This is called the inverse Compton scattering, which is one of the most important processes in high energy astrophysics. Scattering of low energy photons by energetic electrons results in energy gain by the photons and, hence, energy loss for the electrons.

Let us consider the collision between a photon and a relativistic electron. If $\gamma E \ll m_e c^2$, then photon energy in the rest frame of the electron is $E' \ll m_e c^2$. E' is related to E through the Doppler shift formula:

$$E' = \gamma E (1 + \beta \cos\theta)$$

Here, θ is the angle of incidence in the lab frame. In the rest frame of the electron, Compton scattering is simply Thomson scattering (since $E' \ll m_e c^2$), thus resulting in the following radiated power:

$$P' = \sigma_T c U'_{rad}$$

Here, U'_{rad} is the energy density of radiation in the electron rest frame, which is related to U_{rad} according to:

$$U'_{rad} = \left[\gamma (1 + \beta \cos\theta)\right]^2 U_{rad}$$

Averaging over the angle of incidence θ, we find:

$$P' = \frac{4}{3} \sigma_T c U_{rad} \left(\gamma^2 - \frac{1}{4}\right)$$

Since the total power is Lorentz invariant, in the lab frame we have:

$$P = \frac{4}{3} \sigma_T c U_{rad} \left(\gamma^2 - \frac{1}{4}\right)$$

The total Compton power, which is the difference between the radiated power and the incident power, is then found to be:

$$P_{Comp} = \frac{4}{3} \sigma_T c U_{rad} (\gamma^2 - 1) \approx \frac{4}{3} \sigma_T \gamma^2 c U_{rad}$$

We notice the similarity between P_{Comp} and the Synchrotron power $P_{Sync} = \frac{4}{3} \sigma_T c \beta^2 \gamma^2 U_B$ discussed before.

This similarity is not a coincidence. Both of the Compton and Synchrothron emissions are based on the same physical interaction, which is the collision between a charged particle and a photon. Therefore, the power emitted by the charged particle should depend only on the density of photons. In the case of Synchrotron emission the photon density is $u_B = \frac{B^2}{8\pi}$, while in the case of Compton emission it is given by $u_{rad} = \frac{E^2}{4\pi}$.

The similarity between P_{Comp} and P_{sync} provides us with a powerful tool in probing the physical conditions at the high-energy object. For example, if we can measure both a radio (from Synchrotron) and γ-ray (from Compton) spectrum from the same object, then the ratio $\frac{P_{Comp}}{P_{sync}}$ allows us to infer the magnetic field if we know the local u_{rad}, or to probe the ambient photon intensity if we have a measure of the magnetic field (for example, from Farday rotation measurement).

The appearance of γ in P_{Comp} indicates that Compton scattering is an effective process only for highly relativistic electrons. Thus, we are typically led to consider non-thermal particle populations. Even at $T \sim 10^9$ K, a typical electron has a velocity $v \approx \left(\frac{3k}{m_e}\right)^{\frac{1}{2}} \sim \frac{2}{3}c$, with a Lorentz factor $\gamma \sim 1.4$. There are, howev, exceptions like supermassive black holes at the galactic center whose X-ray flux appears to be due to the inverse Compton scattering by a very hot plasma ($T \gg 10^9$ K),

To underline the enormous impact of the Compton power on the emission X-rays and γ-rays, consider the interaction of GeV cosmic ray electrons with the CMB photons in the interstellar medium. The CMB photons have an energy $E_0 \sim 3\times10^{-4}$ eV. Upon scattering off O(GeV) electrons with $\gamma \sim 2\times10^3$, photons with energy $E_{sc} \simeq \gamma^2 E_0 \sim 1$ keV are produced. The scattering therefore converts the microwave photons to X-ray photons.

As mentioned earlier, the majority of Compton sources are non-thermal emitters. To describe them, we consider a power-law distribution:

$$N(\gamma)\, d\gamma = K \gamma^{-n}\, d\gamma$$

Here, we use γ as the independent variable instead of energy E. The total Compton power then is:

$$P_{tot} = \int_{\gamma_{min}}^{\gamma_{max}} P_{Comp}(\gamma)\, N(\gamma)\, d\gamma$$

For a relativistic distribution $\gamma_{min} \approx 1$, which results in:

$$P_{tot} = \frac{4}{3} \sigma_T c U_{rad} \frac{K}{3-n} \left(\gamma_{max}^{3-n} - 1\right) \simeq \frac{4}{3} \sigma_T c U_{rad} \frac{K}{3-n} \gamma_{max}^{3-n}$$

Spectral measurements of a Compton source can provide valuable clues about the physical state of the astrophysical emitters including the spectral index and the high-energy cutoff γ_{max}. A turnover in the spectrum can be an indication of significant cooling process, which delimits the efficiency of particle acceleration, or an evidence that the source has only been active for a short time.